Anelliptic rational approximations of traveltime P-wave reflections in VTI media

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Abstract

Transversely isotropic media with a vertical symmetry axis (VTI) is the model of the subsurface suitable for processing seismic reflection data surface of sedimentary basins formed by shale. The propagation of P waves in VTI media is characterized by four independent elastic parameters and complex algebraic equations for the phase and group velocities. Therefore, there is a need to obtain approximations accurate to the phase and group velocities in VTI homogeneous media.

Several authors have described approaches to the phase and group velocities with only three parameters, in homogeneous horizontally layered VTI media through hypotheses as weak anisotropy of the medium and anellipticity wavefront.

In this work, we have used rational approximants in shifted-hiperbola approximation and obtained rational anelliptitic approximations of the phase and group velocities in homogeneous horizontally stratified VTI media. We have verified the accuracy of the approximations, compared with other approximations in the literature. As an application, we converted the group velocity approximations in nonhyperbolic moveout approximations and performed parameter estimation by means of semblance- based velocity analysis. The results show the validity of anelliptic rational approximations in inverse processes.

Introduction

Due to the limitations of isotropic models in more complex lithologies, such as sedimentary basins formed by shales, the seismic reflection survey is considered as a model of subsurface anisotropic media, especially the VTI media. In homogeneous media TIV, the wavefront of the SH phase velocity is elliptical and has exact equation that depends only two elastic parameters. However, the P- and SV-waves have: strongly anelliptical wavefront for both phase velocity and group velocity; algebraically complicated exact equations for the phase velocity; and are characterized by five independent elastic moduli tensors (c_{ij}). Moreover, even in TIV media, it is difficult to explain exact equations for the group velocity. Another remarkable feature of anisotropic media is the nonhyperbolic behavior of moveout curve. Thus, it is necessary to obtain approximations for the phase and group velocities and thus for moveout curves, which have precision and are practical to perform the steps of seismic data processing.

Thomsen (1986), using the physical characteristics of the elastic parameters and the properties in the vertical direction, introduced a parameter of elastic moduli which facilitates the study of the effects of wave propagation in homogeneous anisotropic media VTI. Alkhalifah and Tsvankin (1995) found that only three of these parameters influence the propagation of P-waves in TIV media. Authors such as Muir and Dellinger (1985), Thomsen (1986), Dellinger (1993), Alkhalifah and Tsvankin (1995), Alkhalifah (1998), Fomel (2004), Psensic (2013), among others, have shown approaches the phase velocity in homogeneous VTI media, which depend explicitly only three elastic parameters.

Dellinger, and Muir (1985) and Dellinger (1993) showed the anelliptical approximation of the phase velocities, using the properties of the elliptical anisotropy. Assuming analogy in the form of approximations obtained elliptical approximation of group velocity and consequently the moveout approximations for TI media. Fowler (2003) defined the elliptical component of the phase velocity, and through convenient parameterization of elastic parameters obtained anelliptical approximation for phase velocity in VTI media, equivalent to those obtained by other authors. However, using heuristics pure, converted them to approaches: by dispersion relations, group velocities and time equations. Fomel (2004) inspired by the anelliptical approximation (Dellinger *et al*,1993) used the shifted-hyperbola approximation (Malovichko 1978; Sword 1987; de Bazelaire 1988; Castle 1994) and Stolt-stretch correction (Stolt 1978; Fomel and Vaillant 2001) to obtain, separately, the "acoustic" phase velocity approximation (Alkhalifah, 1998) of the P-wave in VTI media. However, by analogy form, obtained group velocity approximation and non-hyperbolic traveltime, very accurate.

In this work, we obtained anelliptical approximation for the phase velocity of the compressional wave in vertical media such as rational approximating the shifted-hyperbola approximation (Fomel, 2004). Using the conversion technique by similarity of form (Dellinger, 1993), we obtained anelliptical approximations to group velocity towards these; we obtain new nonhyperbolic moveout approximations. To prove the accuracy of such approximations, we calculated the relative errors of these compared to other approximations. We also conducted semblance-based velocity analysis, to show the robustness of rational approximations of traveltime in estimating parameters.

Phase velocity in VTI media

The wave propagation in VTI media is characterized by the independent elastic parameters, density-normalized:

$a_{11} = v_{Px}^2, a_{33} = v_{Pz}^2, a_{44} = v_{Sz}^2 = v_{Sx}^2, a_{13}$ and a_{66}, where v_{Px} and v_{Sx} are the horizontal velocities of P- and SV-waves, respectively, and v_{Pz} and v_{Sz} are the vertical velocities of P- and SV-waves, respectively.

In TIV media, three distinct waves propagate with phase velocities: v_P (compressional wave), v_{SV} (vertical shear wave) and v_{SH} (horizontal shear wave). The exact equations of these velocities as a function of the phase angle θ are (Gassmann, 1964)

$$v_{SH}^2(\theta) = a_{66}\sin^2\theta + a_{44}\cos^2\theta \tag{1}$$

and

$$v_{P,SV}^2(\theta) = \frac{1}{2}(a_{11}\sin^2\theta + a_{33}\cos^2\theta + a_{44}) \pm$$
$$\pm\tfrac{1}{2}\sqrt{[(a_{11}-a_{44})\sin^2\theta - (a_{33}-a_{44})\cos^2\theta]^2 + (a_{13}+a_{44})^2\sin^2 2\theta} \tag{2}$$

where the signs of addition and subtraction in the radical in (2) are the P- and SV-waves, respectively. Equation (1) is simple and depends only of two elastic parameters, but the equation (2) is algebraically complex and depends explicitly four elastic parameters. This leads to the need to obtain approximations with simpler equations that depend on fewer parameters.

Thomsen (1986) introduced the dimensionless parameters

$$\epsilon = \tfrac{a_{11}-a_{33}}{2a_{33}}, \delta = \tfrac{(a_{13}+a_{44})^2-(a_{33}-a_{44})^2}{2a_{33}(a_{33}-a_{44})} \text{ e } \gamma = \tfrac{a_{66}-a_{44}}{2a_{44}}. \tag{3}$$

to describe TIV media. Alkhalifah and Tsvankin (1995) found that only the parameters ϵ and δ has an effect on P-waves propagating in TIV means and thereby defined the parameter anellipticity

$$\eta = \tfrac{\epsilon-\delta}{1+2\delta}. \tag{4}$$

They showed that this parameter characterizes the anisotropy in such media.

Approximations for P velocities in VTI media

Thomsen (1986) made expansion around the vertical axis and presented an approach to the phase velocity of P-wave to weakly anisotropic media, such as:

$$v_P(\theta) \approx v_{Pz}(1 + \delta\sin^2\theta\cos^2\theta + \epsilon\sin^4\theta). \tag{5}$$

Based on the fact that the group velocity and angle can be obtained, respectively, of the speed and phase angle by:

$$V^2(\Theta) = v^2(\theta) + \left[\tfrac{dv(\theta)}{d\theta}\right]^2 \tag{6}$$

and

$$\tan(\Theta - \theta) = \tfrac{1}{v(\theta)}\tfrac{dv(\theta)}{d\theta}, \tag{7}$$

Thomsen (1986) obtained from (5) to weak anisotropy approximation to group velocity as:

$$V_P(\Theta) \approx v_{Pz}(1 + \delta\sin^2\Theta\cos^2\Theta + \epsilon\sin^4\Theta). \tag{8}$$

However, the weak anisotropy approximation loses accuracy for angles away from the vertical.

Muir e Dellinger (1985) e Dellinger *et al*. (1993) observed that the group and phase velocities in the presence of elliptical anisotropy, has equations that have similar shapes. Thus deduced an anelliptical approximation for P-wave phase velocity in TIV media:

$$v_P^2(\theta) \approx v_{Pe}^2(\theta) + \tfrac{a}{v_{Pe}^2(\theta)}, \tag{9}$$

where $a = [-2\eta/(1+2\eta)]a_{11}a_{33}\sin^2\theta\cos^2\theta$ e $v_{Pe}^2(\theta) \equiv a_{11}\sin^2\theta + a_{33}\cos^2\theta$ is the elliptical contribution to phase velocity. By analogy as deduced also an anelliptical approximation for the P-wave group velocity as:

$$V_P^{-2}(\Theta) \approx V_{Pe}^{-2}(\Theta) + \tfrac{A}{V_{Pe}^{-2}(\Theta)}, \tag{10}$$

where $A = 2\eta a_{11}^{-1}a_{33}^{-1}\sin^2\Theta\cos^2\Theta$ e $V_{Pe}^{-2}(\Theta) \equiv a_{11}^{-1}\sin^2\Theta + a_{33}^{-1}\cos^2\Theta$ is the elliptical contribution to group velocity.

Fomel (2004) interpreted the anelliptic approximation for phase velocity (9) as a linearization, in a (small), the *shifted-hiperbola* approximation (Malovichko, 1978; Sword, 1987; de Bazelaire, 1988; Castle, 1994) given by the function:

$$f(a) = v_{Pe}^2(\theta)(1-s) + s\sqrt{v_{Pe}^4(\theta) + \tfrac{2a}{s}}. \tag{11}$$

Fomel (2004) demonstrated $s = \frac{1}{2}$ and thus obtained an elliptical approximation for P-wave phase velocity in TIV media:

$$v_P^2(\theta) \approx \frac{1}{2}v_{Pe}^2(\theta) +$$
$$+\tfrac{1}{2}\sqrt{v_{Pe}^4(\theta) + 4\left[-\tfrac{2\eta}{1+2\eta}\right]a_{11}a_{33}\sin^2\theta\cos^2\theta} \tag{12}$$

which proved to be equivalent to "acoustic" approach Alkhalifah (1998).

Following the same strategy, Fomel (2004) interpreted the approach of the group velocity (10) as a linearization of the user function:

$$f(A) = V_{Pe}^{-2}(\Theta)(1-S) + S\sqrt{V_{Pe}^{-4}(\Theta) + \tfrac{2A}{S}}. \tag{13}$$

He obtained $S = \frac{1}{4(1+\eta)}$ and, with it, a new and fairly accurate approximation of the group velocity as

$$V_P^{-2}(\Theta) \approx \frac{3+4\eta}{4+4\eta}V_{Pe}^{-2}(\Theta) +$$
$$+\tfrac{1}{4+4\eta}\sqrt{V_{Pe}^{-4}(\Theta) + \left(\tfrac{16\eta}{1+\eta}\right)a_{11}^{-1}a_{33}^{-1}\sin^2\Theta\cos^2\Theta}. \tag{14}$$

Padé approximations for phase and group velocities of P-waves

An alternative to treat complicated algebraic functions is to use polynomial approximations, as the Taylor series. Inherent disadvantages of the approaches by Taylor series, as slow convergence and convergence radius limited to the region of little interest, has led to approaches that provide more information function. Alternatives for such problems are: approximating rational and continued fractions, among others.

A rational approximant, or a Pade approximant, allows to obtain from a power series like a Taylor series, much more information than the very Taylor series. Pade approximant to converge faster and have higher radius of convergence to the Taylor series, and for this, just need the coefficients of the series itself (Baker, 1975).

Consider then the shifted-hyperbola function (11). By expanding it into a Taylor series to the fourth order, rational approximations are obtained: [1,1],[2,1] and [2,2], to phase velocity as

$$v_{P[1,1]}^2(\theta) \approx v_{Pe}^2(\theta)\left(1 + \tfrac{2sa}{2sv_{Pe}^4(\theta)+a}\right) \tag{15}$$

$$v_{P[2,1]}^2(\theta) \approx v_{Pe}^2(\theta)\left[1 + \tfrac{s\,a\,v_{Pe}^4(\theta)+a^2/2}{v_{Pe}^4(\theta)(s\,v_{Pe}^4(\theta)+a)}\right] \tag{16}$$

$$v_{P[2,2]}^2(\theta) \approx v_{Pe}^2(\theta)\left[1+\frac{4\,s\,a(s\,v_{Pe}^4(\theta)+a)}{a^2+6\,s\,a\,v_{Pe}^4(\theta)+4\,s^2 v_{Pe}^8(\theta)}\right] \quad (17)$$

respectively. Making $s=\frac{1}{2}$ (Fomel 2004) obtained anelípticas approximations for P-wave phase velocity in VTI media:

$$v_{P[1,1]}^2(\theta) \approx v_{Pe}^2(\theta)\left(1+\frac{a}{v_{Pe}^4(\theta)+a}\right) \quad (18)$$

$$v_{P[2,1]}^2(\theta) \approx v_{Pe}^2(\theta)\left(1+\frac{v_{Pe}^4(\theta)a+a^2}{v_{Pe}^4(\theta)(\,v_{Pe}^4(\theta)+2a)}\right) \quad (19)$$

$$v_{P[2,1]}^2(\theta) \approx v_{Pe}^2(\theta)\left(1+\frac{v_{Pe}^4(\theta)a+a^2}{a^2+3av_{Pe}^4(\theta)+v_{Pe}^8(\theta)}\right) \quad (20)$$

These approaches have quite simply and present, very explicit, the elliptical and hyperbolic parts. Figure 1 shows the relative error of approximation (5), (9), (12), (18), (19) and (20) shows that the approximations (18), (19) and (20) have excellent accuracy and numerically equivalent to the acoustic approximation (12). They also are much more accurate than the approximation (5) and (9). The model used in this experiment and the next is the TIV Greenhorn shale (Jones and Wang, 1981) which has elastic parameters density-normalized: $a_{11}=14{,}47\,km^2/s^2$, $a_{13}=4{,}51\,km^2/s^2$, $a_{33}=9{,}57\,km^2/s^2$, $a_{55}=2{,}28\,km^2/s^2$.

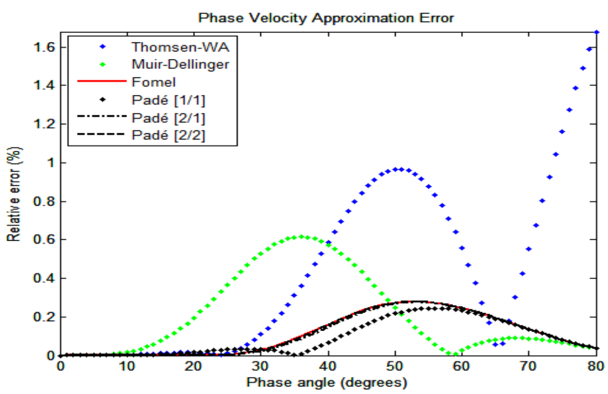

Figure 1. Relative error of the phase velocity approximatios: (5), (9), (12), (18), (19) and (20) in the Greenhorn shale VTI.

Following again, Dellinger and Muir (1985) and Dellinger *et al*. (1993) obtained, heuristically, anelliptical rational approximations for P-wave group velocity in TIV media, with the same shape to the phase velocity approximations (18), (19) and (20):

$$V_{P[1,1]}^{-2}(\Theta) \approx V_{Pe}^{-2}(\Theta)\left(1+\frac{2SA}{2SV_{Pe}^{-4}(\Theta)+A}\right) \quad (21)$$

$$V_{P[2,1]}^{-2}(\Theta) \approx V_{Pe}^{-2}(\Theta)\left[1+\frac{S\,A\,V_{Pe}^{-4}(\Theta)+A^2/2}{V_{Pe}^{-4}(\Theta)(S\,V_{Pe}^{-4}(\Theta)+A)}\right] \quad (22)$$

$$V_{P[2,2]}^{-2}(\Theta) \approx V_{Pe}^{-2}(\Theta)\left[1+\frac{4\,S\,A(S\,V_{Pe}^{-4}(\Theta)+A)}{A^2+6\,S\,A\,V_{Pe}^{-4}(\Theta)+4\,S^2 V_{Pe}^{-8}(\Theta)}\right] \quad (23)$$

Making $S=\frac{1}{4(1+\eta)}$ (Fomel, 2004) obtained elliptical rational approximations for P-wave group velocity in VTI media as:

$$V_{P[1,1]}^{-2}(\Theta) \approx V_{Pe}^{-2}(\Theta)\left(1+\frac{A}{V_{Pe}^{-4}(\Theta)+2(1+\eta)A}\right) \quad (24)$$

$$V_{P[2,1]}^{2}(\Theta) \approx V_{Pe}^{-2}(\Theta)\left[1+\frac{A\,V_{Pe}^{-4}(\Theta)+2(1+\eta)A^2}{V_{Pe}^{-4}(\Theta)(V_{Pe}^{-4}(\Theta)+4(1+\eta)A)}\right] \quad (25)$$

$$V_{P[2,2]}^{2}(\Theta) \approx V_{Pe}^{-2}(\Theta)\left[1+\frac{A(\,V_{Pe}^{-4}(\Theta)+4(1+\eta)A)}{4(1+\eta)^2A^2+6\,(1+\eta)\,A\,V_{Pe}^{-4}(\Theta)+V_{Pe}^{-8}(\Theta)/4}\right] \quad (26)$$

These approaches also have very easily with elliptical and anelíptica parts well defined. Figure (2) shows the relative error of approximation (8), (10) (14) (24) (25) and (26). This shows the excellent accuracy of the approximations (24), (25) and (26) and their validity by comparing with the approaches (8), (10) and (14).

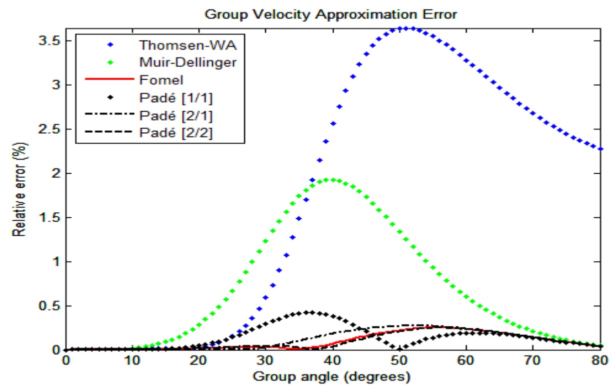

Figure 2. Relative error of the group velocity approximations (8), (10), (14), (24), (25) and (26) in Greenhorn shale VTI.

Moveout Approximatios

In CMP gathers, a reflected P-wave in a homogeneous medium VTI has traveltime equation as a function of offset *x* given by:

$$t^2(x) = \frac{4z^2+x^2}{V_P^2(\Theta)}, \quad (27)$$

where $\Theta=\tan^{-1}\left(\frac{x}{t_z v_{Pz}}\right)$, with $t_{Pz}=2z/v_{Pz}$ the two-way vertical traveltim. Thus, it is possible to obtain by simple replacement of the group velocity approximation (8), (10), (14), (24), (25) and (26) into equation (27) nonhyperbolic moveout approximations.

The approximation to nonhyperbolic traveltime, related the group velocity approximation (10) is given by (Dellinger *et al*., 1993):

$$t_P^2(x) \approx t_{Pz}^2+\frac{x^2}{v_{Pn}^2}+\frac{(v_{Pn}^2-v_{Px}^2)x^4}{v_{Pn}^2 v_{Px}^2(v_{Pn}^2 t_{Pz}^2-x^2)}. \quad (28)$$

where $v_{Pn}^2=v_{Px}^2/(1+2\eta)$ is the *normal-moveout* (NMO) velocity to interface without dip.

Since the non-hyperbolic approximation time related to the group velocity approximation (14) is given by: (Fomel, 2004)

$$t_P^2(x) \approx \frac{3+4\eta}{4+4\eta}t_{Ph}^2(x)+\frac{1}{4+4\eta}\sqrt{t_{Ph}^4(x)+\left[\frac{16\eta(1+\eta)}{(1+2\eta)}\right]\frac{t_{Pz}^4x^2}{v_{Pn}^2}}. \quad (29)$$

where $t_{Ph}^2(x)=t_{Pz}^2+x^2/[(1+2\eta)v_{Pn}^2]$ is the hyperbolic contribution in moveout equation.

Finally, non-hyperbolic moveout approximations relating to the group velocity approximation (24), (25) and (26) has, respective shapes

$$t_P^2(x) \approx t_{Ph}^2(x)\left(1+\frac{1}{\frac{(1+2\eta)v_{Pn}^2 t_{Ph}^4(\theta)}{2\eta t_{Pz}^2 x^2}+2(1+\eta)}\right), \quad (30)$$

$$t_P^2(x) \approx t_{Ph}^2(x)\left(1+\frac{(1+2\eta)v_{Pn}^2 t_{Ph}^4(\theta)+4\eta t_{Pz}^2 x^2(1+\eta)}{(1+2\eta)v_{Pn}^2 t_{Ph}^4(\theta)\left[\frac{(1+2\eta)v_{Pn}^2 t_{Ph}^4(\theta)}{2\eta t_{Pz}^2 x^2}+4(1+\eta)\right]}\right) \qquad (31)$$

e

$$t_P^2(x) \approx t_{Ph}^2 \times \left(1+\frac{(1+2\eta)v_{Pn}^2 t_{Ph}^4(\theta)+8\eta t_{Pz}^2 x^2(1+\eta)}{(1+2\eta)v_{Pn}^2 t_{Ph}^4(\theta)\left[\frac{(1+2\eta)v_{Pn}^2 t_{Ph}^4(\theta)}{2\eta t_{Pz}^2 x^2}+6(1+\eta)\right]+8\eta t_{Pz}^2 x^2(1+\eta)^2}\right) \qquad (32)$$

It is worth mentioning here, the traveltime approximation (Alkhalifah and Tsvankin, 1995) given by:

$$t_P^2(x) \approx t_{Pz}^2+\frac{x^2}{v_{Pn}^2}+\frac{2\eta x^4}{v_{Pn}^2\left(v_{Pn}^2 t_{Pz}^2+(1+2\eta)x^2\right)}. \qquad (33)$$

Figure (3) shows the relative error of the approximations (28) (29) (30) (31) (32) and (33). As might be expected, the rational approximations (30), (31), (32) have level of accuracy in the order of milliseconds and is equivalent to the approximation (29). They also have the lowest absolute error approximations (28) and (33).

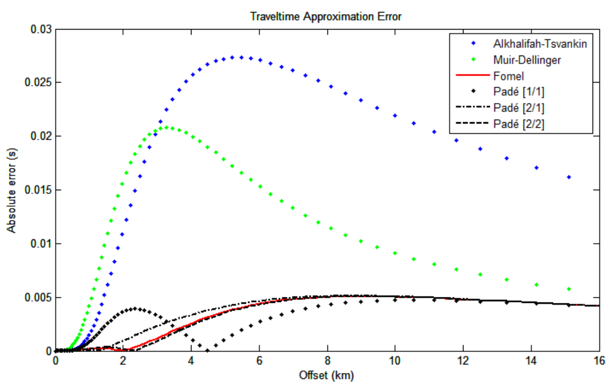

Figure 2. Relative error of the moveout approximations in Greenhorn shale VTI.

Application in Parameter Estimation.

Velocity analysis by semblance in a CMP (*common midpoint*) gather, is traditionally performed to estimate t_{Pz} and v_{Pn} maximizing functional (Tarner and Koehler, 1969):

$$S(t'_{Pz}, v)=\frac{\sum_{t_1}\left[\sum_x D_v(t_1,x)\right]^2}{N\sum_{t_1}\sum_x D_v(t_1,x)}, \qquad (25)$$

where N is the number of traces; t_1 are zero-offset traveltimes in a time window centered on t'_{Pz}; $D_v(t_1,x)=D[t_v(t_1,x),x]$ the data sampled on a moveout curve calculated for a given velocity v with $t_v^2(t_1,x)=t_1^2+(x^2/v^2)$. However, this operator is limited to offset-depth ratio at most 1.

Because the effects of anisotropy in CMP gathers, are more prominent in long offsets, Alkhalifah (1997) adapted the semblance to TIV media introducing non-hyperbolic moveout curve (33) in order to estimate the parameters t_{Pz}, v_{Pn} e η. Grechka and Tsvankin (1998) observed instability in non-hyperbolic semblance of Alkhalifah (1997) to reverse the parameter η. Thus, added a correction factor in (33), reescrevemdo it on account of the parameters: t_{Pz}, v_{Pn} e v_{Px} for the estimate.

In this experiment, we perform velocity analysis using the semblance of nonhyperbolic (Grechka and Tsvankin,1998) with nonhyperbolic moveout equations (30), (31) and (32), presented herein. Synthetic CMP section in Figure 4, was drawn to the TIV material Greenhorn shale with reflecting interface to the depth $z=1$. Traveltime is accurate and the source of signing a Ricker pulse with dominant frequency $f=20Hz$.

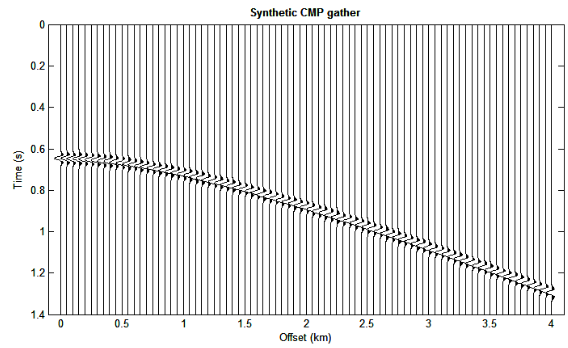

Figure 4. Synthetic CMP obtained for Greenhorn shale VTI.

To illustrate the velocity analysis, Figure 5 shows the semblance map v_{Pn} *versus* v_{Px} obtained in velocity analysis using the nonhyperbolic moveout equation (31). The exact and estimated values of the velocities v_{Pn} and v_{Px}, and parameter η, and the relative error are presented in Table 1. The errors presented show the robustness of these approximations in the estimate parameters in VTI media.

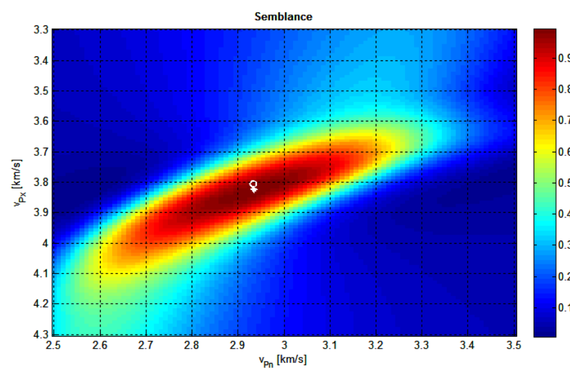

Figure 5. Map semblance as a function of v_{Pn} and v_{Px} velocitiess in Greenhorn shale VTI.

Table 1. Valores exatos e estimados dos parâmetros v_{Pn}, v_{Px} e η na análise de velocidades.

	$v_{Pn}(km/s)$	$v_{Px}(km/s)$	η
Valor Exato	2.9336	3.8045	0.3409
Valor Aprox.	2.9343	3.8253	0.3497
Erro Relativo (%)	0.0255	0.5458	2.5726

Figures 6, 7 and 8 show, for various values of the offset-depth ratio, the corresponding estimated values of the velocities v_{Pn} and v_{Px}, and parameter η, using the moveout approximations obtained here, as compared to the moveout approximations (29) and (33). The maximum semblance values are shown in Figure 9. The rational approximation [1,1] proved to be excellent to estimate parameters for values of offset-depth ratio up to 3, and the rational approximation [2,1] for large offsets.

Conclusions

We obtained anelliptical rational approximations of the phase and group velocities in homogeneous VTI media, horizontally stratified. Here, too, new and accurate traveltime approximations. The knowledge of these new approaches, following the technique of obtaining anelliptics approximations, by similarity, and shifted-hyperbola curve. We checked, by the relative error calculation showed that these new approaches become quite accurate. We also verified the validity of such comparing them with the literature known approaches. To show the robustness of moveout approximations, we estimate parameters by based-semblance velocity analysis. The results showed that they are good for estimating parameters in the inverse process.

Acknowledgments

The UFPa for support at all times of the research.

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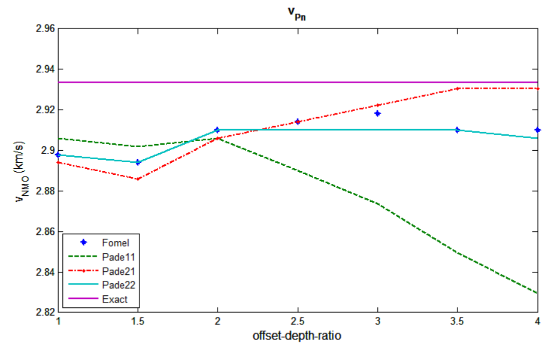

Figura 6. Estimated values of v_{Pn} obtained from velocity analysis with the traveltime approximations (29), (30), (31), (32) and (33) in the Greenhorn shale VTI.

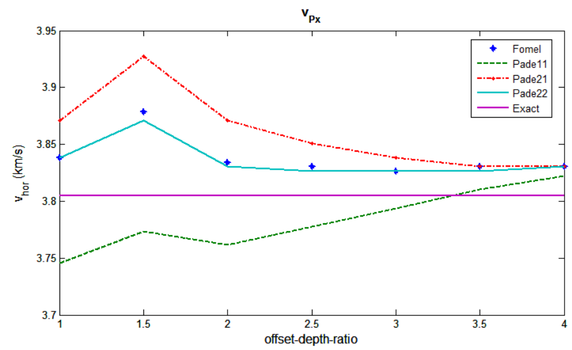

Figure 7. Estimated values of v_{Pz} obtained from velocity analysis with the traveltime approximations (29), (30), (31), (32) e (33) in the Greenhorn shale VTI.

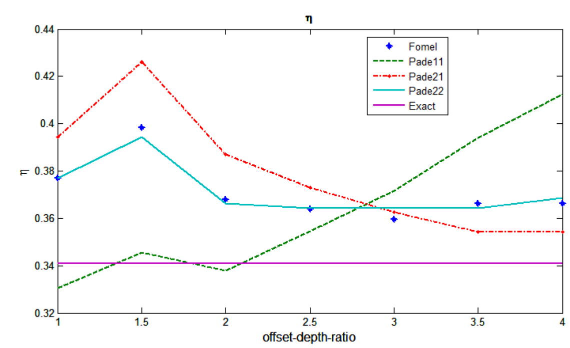

Figure 8. Estimated values of η obtained from velocity analysis with the traveltime approximations (29), (30), (31), (32) e (33) in the Greenhorn shale VTI.

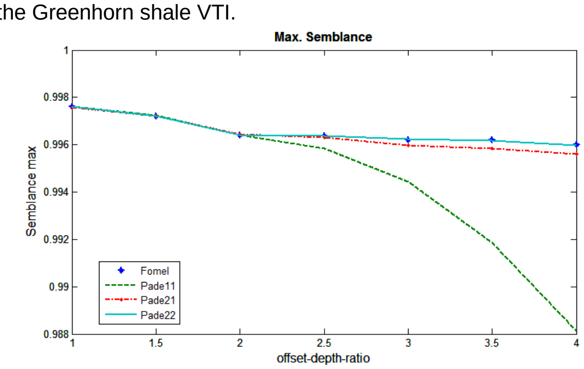

Figure 9. Maximum semblance values obtained from velocity analysis with the traveltime approximations (29), (30), (31), (32) e (33) in the Greenhorn shale VTI.